Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MD01

Unit Decision 1

Thursday 24 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

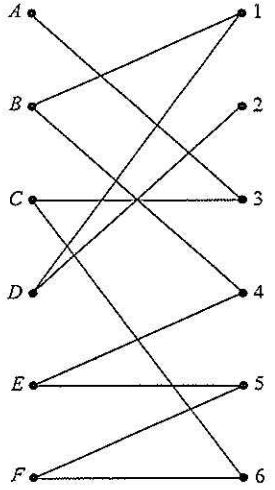
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 Six people, A, B, C, D, E and F, are to be allocated to six tasks, 1, 2, 3, 4, 5 and 6. The following bipartite graph shows the tasks that each of the people is able to undertake.



(a) Represent this information in an adjacency matrix. *(2 marks)*

(b) Initially, B is assigned to task 4, C to task 3, D to task 1, E to task 5 and F to task 6.

By using an algorithm from this initial matching, find a complete matching. *(3 marks)*

QUESTION PART REFERENCE

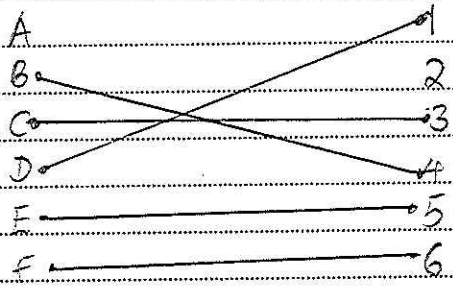
Answer space for question 1

	1	2	3	4	5	6
(a) A	0	0	1	0	0	0
B	1	0	0	1	0	0
C	0	0	1	0	0	1
D	1	1	0	0	0	0
E	0	0	0	1	1	0
F	0	0	0	0	1	1

QUESTION PART REFERENCE

Answer space for question 1

(b)



Complete matching

A – 3 – C

A – 3 – C – 6 – F – 5 – E – 4 – B – 1 – D – 2

Match

A – 3 – C – 6 – F – 5 – E – 4 – B – 1 – D – 2

A 3, B 1, C 6, D 2, E 4, F 5

2 A student is using a shuttle sort algorithm to rearrange a set of numbers into ascending order.

Her correct solution for the first three passes is as follows.

Initial list	10	7	4	22	23	26
After 1st pass	7	10	4	22	23	26
After 2nd pass	4	7	10	22	23	26
After 3rd pass	4	7	10	22	23	26

(a) Write down the number of comparisons on each of the three passes. *(2 marks)*

(b) Write down the number of swaps on each of the three passes. *(2 marks)*

(c) Explain whether or not the student has completed the algorithm. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

(a) 1st 1 comparison

2nd 2 comparisons

3rd 1 comparison

(b) 1st 1 swap [10 7]

2nd 2 swaps [10 4] [7 4]

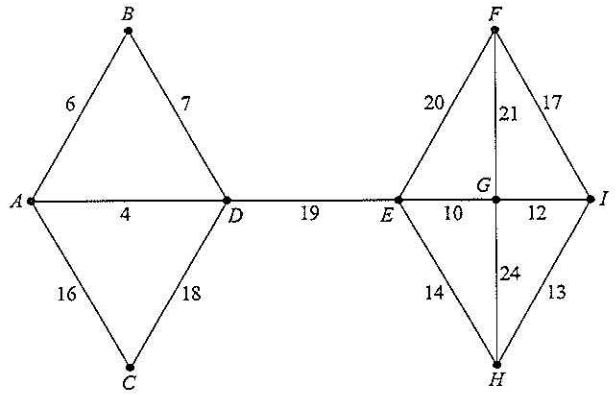
3rd no swaps.

(c) No, they have to check 23 and 26

QUESTION PART REFERENCE

Answer space for question 2

3 The following network shows the lengths, in miles, of roads connecting nine villages, $A, B, \ldots, I$.



(a) (i) Use Prim's algorithm starting from A, showing the order in which you select the edges, to find a minimum spanning tree for the network. *(4 marks)*

(ii) State the length of your minimum spanning tree. *(1 mark)*

(iii) Draw your minimum spanning tree. *(2 marks)*

(b) Prim's algorithm from different starting points produces the same minimum spanning tree for this network. State the final edge that would complete the minimum spanning tree using Prim's algorithm:

(i) starting from D: *(1 mark)*

(ii) starting from H. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 3

(a)

(i)

AD 4

AB 6

AC 16

DE 19

EG 10

GI 12

IH 13

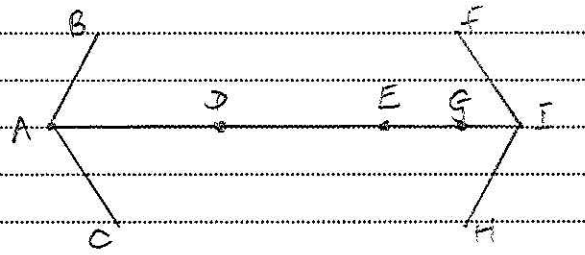
IF 17

QUESTION PART REFERENCE

Answer space for question 3

(i) length = 97

(ii)



b

(i) IF

(ii) AC

4 The edges on the network below represent some major roads in a city. The number on each edge is the minimum time taken, in minutes, to drive along that road.

(a) (i) Use Dijkstra's algorithm on the network to find the shortest possible driving time from A to J. *(5 marks)*

(ii) Write down the corresponding route. *(1 mark)*

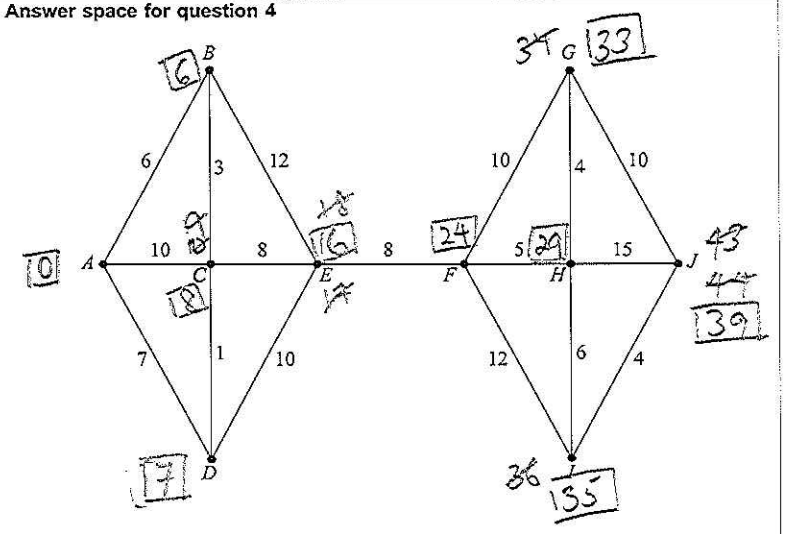
(b) A new ring road is to be constructed connecting A to J directly.

Find the maximum length of this new road from A to J if the time taken to drive along it, travelling at an average speed of 90 km/h, is to be no more than the time found in part **(a)(i)**. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 4

(a)(i)



(ii) A D C E F H I J

(b) A – J => 39 minutes

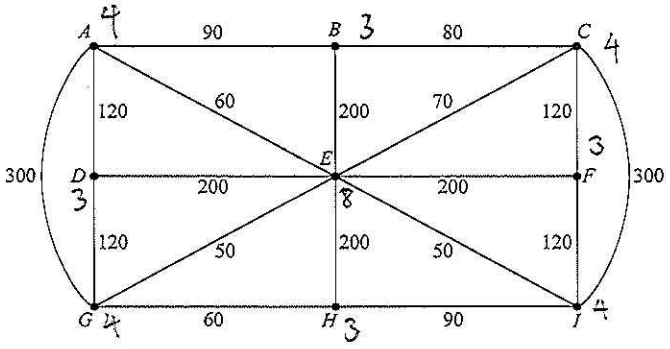
$\frac{39}{60} \times 90 = 58.5$ km

QUESTION PART REFERENCE

Answer space for question 4

5 The network below shows some streets in a town. The number on each edge shows the length of that street, in metres.

Leaflets are to be distributed by a restaurant owner, Tony, from his restaurant located at vertex B. Tony must start from his restaurant, walk along all the streets at least once, before returning to his restaurant.



The total length of the streets is 2430 metres.

(a) Find the length of an optimal Chinese postman route for Tony. *(5 marks)*

(b) Colin also wishes to distribute some leaflets. He starts from his house at H, walks along all the streets at least once, before finishing at the restaurant at B.

Colin wishes to walk the minimum distance. Find the length of an optimal route for Colin. *(1 mark)*

(c) David also walks along all the streets at least once. He can start at any vertex and finish at any vertex. David also wishes to walk the minimum distance.

(i) Find the length of an optimal route for David. *(1 mark)*

(ii) State the vertices from which David could start in order to achieve this optimal route. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 5

(a) Odds → B D F H

BD + FH = 210 + 210 = 420

BF + DH = 200 + 180 = 380 ✓

BH + DF = 260 + 340 = 600

QUESTION PART REFERENCE

Answer space for question 5

length = 2430 + 380
= 2810.

(b) 2430 + DF = 2430 + 340
= 2770

(c)

(i) 2430 + DH = 2430 + 180
= 2610

(ii) B and F only

6 The complete graph K_n $(n > 1)$ has every one of its n vertices connected to each of the other vertices by a single edge.

(a) Draw the complete graph K_4. *(1 mark)*

(b) (i) Find the total number of edges for the graph K_8.

(ii) Give a reason why K_8 is not Eulerian. *(2 marks)*

(c) For the graph K_n, state in terms of n:

(i) the total number of edges;

(ii) the number of edges in a minimum spanning tree;

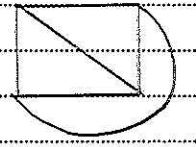
(iii) the condition for K_n to be Eulerian;

(iv) the condition for the number of edges of a Hamiltonian cycle to be equal to the number of edges of an Eulerian cycle. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 6

6

(a) 

(b)

(i) $K_8 = \frac{8(8-1)}{2} = \frac{8 \times 7}{2} = \frac{56}{2}$
$= 28$

(ii) Odd number of edges at all vertices.

(c)

(i) total number of edges $\frac{n(n-1)}{2}$

QUESTION PART REFERENCE

Answer space for question 6

(ii) $n-1$

(iii) n must be odd

(iv) $n = 3$

7 Rupta, a sales representative, has to visit six shops, *A*, *B*, *C*, *D*, *E* and *F*. Rupta starts at shop *A* and travels to each of the other shops once, before returning to shop *A*. Rupta wishes to keep her travelling time to a minimum.

The table shows the travelling times, in minutes, between the shops.

	A	*B*	*C*	*D*	*E*	*F*
A		16	10	25	26	40
B	16		20	19	18	50
C	10	20	–	14	22	31
D	25	19	14		11	32
E	26	18	22	11	–	42
F	40	50	31	32	42	

(a) Find the travelling time of the tour *ACFDEBA*. *(1 mark)*

(b) Use the nearest neighbour algorithm, starting at *A*, to find an upper bound for the travelling time for Rupta's tour. *(4 marks)*

(c) By deleting *A*, find a lower bound for the travelling time for Rupta's tour. *(4 marks)*

(d) Sketch a network showing the edges that give you the lower bound in part **(c)** and comment on its significance. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 7

(a) A C F D E B A

$10 + 31 + 32 + 11 + 18 + 16 = 118$

118.

(b) A C D E B F A

10 14 11 18 50 40

= 143 (added together)

QUESTION PART REFERENCE

Answer space for question 7

16 B

A

10 C

16 + 10 = 26

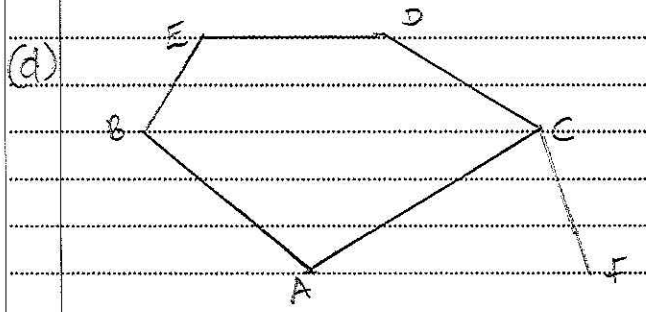
	B 1	C 4	D 3	E 2	F 5
B	—	20	14	18	50
C	20	—	(14)	22	31
D	19	14		(11)	32
E	(18)	22	11	—	42
F	50	(31)	32	42	—

B E D C F

18 + 11 + 14 + 31 = 74

Total = 26 + 74 = 100

(d)



The lower bound does not make a cycle and bur is greater than 100.

8 The following algorithm finds an estimate of the value of the number represented by the symbol e:

Line 10 Let $A = 1, B = 1, C = 1$
Line 20 Let $D = A$
Line 30 Let $C = C \times B$
Line 40 Let $D = D + (1/C)$
Line 50 If $B = 4$ then go to Line 80
Line 60 Let $B = B + 1$
Line 70 Go to Line 30
Line 80 Print 'An estimate of e is', D
Line 90 End

(a) Trace the algorithm. *(6 marks)*

(b) A student miscopied Line 70.

His line was

Line 70 Go to Line 10

Explain what would happen if his algorithm were traced. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 8

(a)

A	B	C	D
1	1	1	1
			2
		2	2
	2	2	2.5
3	6	6	2.67
4	24	24	2.71

QUESTION PART REFERENCE

Answer space for question 8

(b) It would create a never-ending loop. A, B, C always reset to 1.

9 Ollyin is buying new pillows for his hotel. He buys three types of pillow: soft, medium and firm.

He must buy at least 100 soft pillows and at least 200 medium pillows.

He must buy at least 400 pillows in total.

Soft pillows cost £4 each. Medium pillows cost £3 each. Firm pillows cost £4 each.

He wishes to spend no more than £1800 on new pillows.

At least 40% of the new pillows must be medium pillows.

Ollyin buys x soft pillows, y medium pillows and z firm pillows.

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, find five inequalities in x, y and z that model the above constraints. *(3 marks)*

(b) Ollyin decides to buy twice as many soft pillows as firm pillows.

(i) Show that three of your answers in part **(a)** become

$$3x + 2y \geqslant 800$$

$$2x + y \leqslant 600$$

$$y \geqslant x$$

(3 marks)

(ii) On the grid opposite, draw a suitable diagram to represent Ollyin's situation, indicating the feasible region. *(5 marks)*

(iii) Use your diagram to find the maximum total number of pillows that Ollyin can buy. *(2 marks)*

(iv) Find the number of each type of pillow that Ollyin can buy that corresponds to your answer to part **(b)(iii)**. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 9

(a) $x \geqslant 100$, $y \geqslant 200$

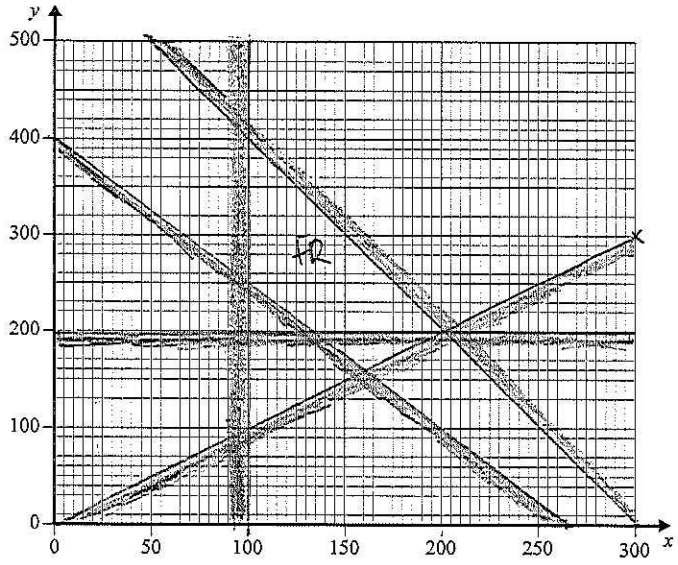
$x + y + z \geqslant 400$

$4x + 3y + 4z \leqslant 1800$

$y \geqslant \frac{40}{100}(x + y + z)$

QUESTION PART REFERENCE (b)(ii)

Answer space for question 9



(b)

(i) $x = 2y \Rightarrow \frac{1}{2}x = y$

$x + y + \frac{1}{2}x \geq 400$

$2x + y + x \geq 800$

$3x + y \geq 800$

$4x + 3y + 2x \leq 1800$

$6x + 3y \leq 1800$

$\div 3 \quad 2x + y \leq 600$

Turn over ▶

QUESTION PART REFERENCE

Answer space for question 9

$5y \geq 2x + 2y + 3x$

$3y \geq 3x$ (−2y both sides)

$\div 3 \quad y \geq x \quad \div 3$

(ii) maximum $y + \frac{3}{2}x$

$400 + 150 = 550$

(iv) Ollyin buys 100 soft, 400 medium and 50 firm pillows.

END OF QUESTIONS